

Applicants for winter semester 2019/20 must select one of the following essay themes:

If you have any questions please do not hesitate to contact us via e-mail: topmath@ma.tum.de.

1. Erdős-Rényi graphs

The Erdős-Rényi graph is the simplest model of random graph, and yet exhibits rich behavior. The model is defined as follows: We choose two parameters, a positive integer n and a real number $0 \leq p \leq 1$. Then the random graph $G_{n,p}$ is the following graph: The set of vertices is $\{1, 2, \dots, n\}$, and for every $i < j$ an edge between i and j exists with probability p . The events of edge existence are independent of each other.

We let $R(G_{n,p})$ be the size of the largest connected component in $G_{n,p}$. Note that this is a random variable. We fix now a constant c , and consider the behavior of the random variable $R_c(n) := R(G_{n,c/n})$.

A famous theorem of Erdős and Rényi states that for every c there is a constant $\phi(c)$ s.t. for every ϵ ,

$$\lim P[|R_c(n)/n - \phi(c)| < \epsilon] = 1.$$

Furthermore, if $c < 1$ then $\phi(c) = 0$ and if $c > 1$ then $\phi(c) > 0$. This is one of the simplest known instances of a phase transition.

In the essay, you should explain the ideas behind the proof of this theorem, and give full details for the $c < 1$ case. You can continue to explain what the asymptotic size of the largest component in the $c < 1$ case is, or show that the second largest class is of size $o(n)$ for all choices of c .

2. Symmetric polynomials

A polynomial in variables x_1, \dots, x_n is called *symmetric* if it remains unchanged under every permutation of the variables. The main theorem on symmetric polynomials states that every symmetric polynomial is uniquely a polynomial in the so-called *elementary symmetric polynomials*.

In this essay, you should state this theorem and prove it (or sketch a proof). Then, you may give examples, or you could explain algorithms that trans-

form every given symmetric polynomial into this unique polynomial in the elementary symmetric polynomials. Another direction of the essay could be the application to discriminants of polynomials, which are symmetric polynomials in the zeros of a given polynomial.

3. Period three implies chaos

Assume that a continuous self-map of the interval has a periodic point of period three. If additionally a simple condition on this point is met, then the map has periodic points of every period $k \in \mathbb{Z}$.

In your essay, you shall state this famous theorem of Li and Yorke and prove it. You may further point out generalizations or relate this result to various notions of ‘chaos’ in dynamical systems. You may also describe algorithms for computing periodic orbits of or up to a given period.

4. The Euler-Maclaurin Sum Formula

The difference between an equidistant sum of a given function and the corresponding definite integral can be expressed in terms of a (divergent) series given by the famous Euler-Maclaurin sum formula.

In this essay you should discuss the analytic properties of this series (divergence, remainder estimates, Leibniz semi-convergence) and illustrate its usefulness by means of a couple of interesting applications taken, e.g., from the following list:

- (1) highly accurate calculation of the Euler-Mascheroni constant or more general Euler-Maclaurin constants,
- (2) Riemann Zeta function and its functional equation,
- (3) Ramanujan summation of divergent series,
- (4) Romberg quadrature and extrapolation methods with order and step-size control in numerical integration,
- (5) renormalization.