Critical Transitions and Nonautonomous Bifurcations

TUM Science & Study Center of the
Technical University Munich in Raitenhaslach
24 - 26 August 2022

Program and Abstracts

Wednesday 24.08.2022

09:00	Shuttle Hotel – Raitenhaslach
09:30 – 10:00	Registration
10:00 – 10:50	P. Kloeden
10:50 – 11:20	Coffee break
11:20 – 12:10	C. Jones
12:10 – 14:00	Lunch break
14:00 – 14:50	T. Caraballo
14:50 – 15:20	X. Jiaohui
15:20 – 16:00	Coffee break
16:00	Shuttle Raitenhaslach – Hotel
16:30 – 18:00	Guided tour of Burghausen's castle

Thursday 25.08.2022

09:15	Shuttle Hotel – Raitenhaslach
10:00 – 10:50	R. Obaya
10:50 – 11:20	Coffee break
11:20 – 12:10	C. Poetzsche
12:10 – 14:00	Lunch break
14:00 – 14:50	C. Nunez
14:50 – 15:20	J. Dueñas
15:20 – 16:00	Coffee break
16:00 – 16:30	K. Slyman
16:45	Shuttle Raitenhaslach – Hotel
19:00	Workshop dinner at Bichl

Friday 26.08.2022

08:45	Shuttle Hotel – Raitenhaslach
09:30 – 10:20	A. Cherubini
10:20 – 10:50	Coffee break
10:50 – 11:40	M. Engel
11:40 – 12:10	D. Chemnitz
12:10 – 14:00	Lunch break
14:15	Shuttle Raitenhaslach – Hotel

Wednesday 24.08.2022

Nonautonomous and stochastic bifurcations: a personal history.

Peter Kloeden – Universität Tübingen
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I visited Ludwig Arnold in Bremen in the early 1980s to talk about stochastic bifurcations. He seemed the natural person to ask since he and his group had been looking at stability and loss of stability in linear stochastic systems. We worked hard on what we thought was a straightforward basic problem and got nowhere, and then turned to other things such as stochastic attractors and stochastic numerics, but always with stochastic bifurcations in the back of our minds. I will discuss the above failed attempt, related ideas and work by myself and others during the twenty year period 1990-2010 on bifurcations in both stochastic and nonautonomous deterministic systems, which have many features in common.

Interactions of Noise and Transient Dynamics that Facilitate Tipping

Christopher K. Jones – University of North Carolina, Chapel Hill
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Analyses of noise-induced tipping have been dominated by Freidlin-Wentzell (F-W) theory. This theory has its roots in the study of systems with underlying deterministic dynamics with a gradient vector field. F-W generalized the key results to non-gradient systems by introducing the concept of a quasipotential. Although popularly derived as a solution to a Hamilton-Jacobi equation, the quasipotential can also be viewed in terms of the projection of an unstable manifold in the Hamiltonian system derived from the Euler-Lagrange equations for minimizing the action functional. A shortcoming of the F-W theory is that it requires vanishingly small noise and for many applications of interest, particularly in social, biological or environmental contexts, intermediate levels of noise are more appropriate. At such intermediate noise levels, the transient behavior of the underlying deterministic system becomes relevant, and I will explore how the above-mentioned dynamical systems viewpoint of the quasipotential can shed light on the effect of this interaction on tipping. The talk will center around two examples: (1) Escape through an unstable periodic orbit due to noise, and (2) The interplay of rate and noise induced tipping. The latter example will be elaborated upon in the companion talk by Katie Slyman.

Asymptotic behavior of a semilinear problem in heat conduction with long time memory and non-local diffusion

Tomas Caraballo – Universidad de Sevilla
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In this talk, the asymptotic behavior of a semilinear heat equation with long time memory and non-local diffusion is analyzed in the usual set-up for dynamical systems generated by differential equations with delay terms. This approach is different from the previous published literature on the long time behavior of heat equations with memory which is carried out by the Dafermos transformation. As a consequence, the obtained results provide complete information about the attracting sets for the original problem, instead of the transformed one. In particular, the proved results also generalize and complete previous literature in the local case (see [1], [2]). This is a joint work with J. Xu and J. Valero.

References:
1. J. Xu, T. Caraballo, J. Valero. Asymptotic behavior of a semilinear problem in heat conduction with long time memory and non-local diffusion, J. Differential Equations 327 (2022), 418-447
2. J. Xu, T. Caraballo, J. Valero. *Asymptotic behavior of nonlocal partial differential equations with long time memory.* Discrete and Continuous Dynamical Systems, Series S (2022) (to appear).

Dynamics of stochastic nonlocal reaction-diffusion equations driven by multiplicative noise

Jiaohui Xu – Universidad de Sevilla
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This paper deals with fractional stochastic nonlocal partial differential equations driven by multiplicative noise. We first prove the existence and uniqueness of solution to this kind of equations with white noise by applying the Galerkin method. Then, the existence and uniqueness of tempered pullback random attractor for the equation are ensured in an appropriate Hilbert space. When the fractional nonlocal partial differential equations are driven by colored noise, which indeed are approximations of the previous ones, we show the convergence of solutions of Wong-Zakai approximations and the upper semicontinuity of random attractors of the approximate random system as the covariance parameter goes to zero.

Guided tour of Burghausen's castle

Meeting point at Curaplatz (3 minutes walk from Hotel Glöcklhofer) at 16:30 sharp.

Thursday 25.08.2022

Uniform stability and chaotic dynamics in nonhomogeneous linear dissipative scalar ODEs

Rafael Obaya – Universidad de Valladolid
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This talk analyzes the structure and the inner long-term dynamics of the invariant compact sets for the skew-product flow induced by a family of time-dependent ordinary differential equations of nonhomogeneous linear dissipative type. The main assumptions are made on the dissipative term and on the homogeneous linear term of the equations. The rich range of possible dynamics scenarios includes the uniform stability of the invariant compact sets, as well as the presence of Li-Yorke chaos and Auslander-Yorke chaos inside the attractor. This is a joint work with Juan Campos and Carmen Nuñez.

Nonautonomous solution bifurcation: Old and new results!

Christian Pötzsche – Universität Klagenfurt
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Nonhyperbolicity i.e. the lack of an exponential dichotomy, is a necessary condition for the bifurcation of bounded (or homoclinic) solutions. One possibility to create such a nonhyperbolic situation is to merge equations being hyperbolic on the positive and negative semi axis individually. Based on this assumption, we discuss several sufficient criteria for the bifurcation of bounded solutions. They are of analytical and of topological nature.

Critical transitions in piecewise uniformly continuous concave quadratic ordinary differential equations

Carmen Núñez – Universidad de Valladolid
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In the analysis of a concave quadratic scalar ordinary differential equation, an already classical approach to the idea of critical transition is to consider that it occurs when a small variation of the coefficients changes completely the global dynamics, from the existence of an attractor-repeller pair of hyperbolic solutions to the lack of bounded solutions. In this work, we describe a tool to determine the occurrence of these dramatic variations, and use it to determine the occurrence of critical transitions for parametric equations of a certain type: asymptotically nonautonomous ODEs with bounded uniformly continuous or bounded piecewise uniformly continuous coefficients. Some numerical experiments contribute to clarify the applicability of this tool. This is a joint work with Iacopo P. Longo and Rafael Obaya.

Bifurcations and critical transitions in scalar nonautonomous differential equations with concave derivative

Jesús Dueñas Pamplona – Universidad de Valladolid
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he structure of minimal sets of some one-parametric bifurcation prob- lems of dissipative nonautonomous scalar ordinary differential equations with con- cave derivative is studied by means of the skewproduct formalism. Different types of bifurcation points of minimal sets are found: saddle-node, transcritical and pitch- fork. These bifurcations provide an adequate framework to study critical transi- tions: sudden and abrupt changes in the state of a complex system which occur on account of small variations on external parameters.

Rate and Noise Tipping Working in Concert

Katherine Slyman – University of North Carolina, Chapel Hill
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We present a theory for understanding tipping events in low-dimensional dynamical systems with additive noise and time-dependent parameters, whose interplay results in a large increase in the frequency of tipping. While rate-induced tipping does not require any random fluctuations within the system, the ramp parameter and added noise can conspire to cause tipping of the system below the critical rate. Building on the work of Ritchie and Sieber (2016), who considered rates close to the critical rate, we first consider a one-dimensional differential equation with additive noise and a ramp parameter. In this model, using the Fredlin-Wentzell theory, we show that there exists a heteroclinic connection in extended phase space between equilibria for all rates less than the critical rate. This heteroclinic orbit is a minimizer of the Freidlin-Wentzell functional and thus corresponds to the most probable path between these two points. We then extend this framework to show the existence of the most probable path for a fairly general class of functions. We construct this most probable path using geometric dynamical systems methods, as well as present numerical simulations for verification and visualization of this most probable path.

Workshop dinner at Bichl - Café Bar Restaurant

Meeting point at In den Grüben 162, 84489 Burghausen (15 minutes walk from Hotel Glöcklhofer) at 19:00.

Friday 26.08.2022

Energy drift in randomly perturbed Hamiltonian systems

Anna Maria Cherubini – Università del Salento
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It is well known that, generically, integrable Hamiltonian systems subjected to small, time-dependent perturbations, generate some orbits that experience significant energy growth. We study the effect of random time-dependent perturbations on integrable Hamiltonian systems, in order to extend results on the 'Arnold diffusion problem' to the context of random dynamical systems. This work is in collaboration with Marian Gidea.

The conditioned Lyapunov spectrum for random dynamical systems

Maximilian Engel – Freie Universität Berlin
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We establish the existence of a full spectrum of Lyapunov exponents for memoryless random dynamical systems with absorption. To this end, we embed the process conditioned to never being absorbed, the Q-process, into the framework of random dynamical systems, allowing us to study multiplicative ergodic properties. We show that the finite-time Lyapunov exponents converge in conditioned probability and apply our results to iterated function systems and stochastic differential equations, making the results relevant for random bifurcation problems.

New results on Shear Induced Chaos in the Hopf Normal Form with Additive Noise

Dennis Chemnitz – Freie Universität Berlin
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We are concerned with the problem of finding positive Lyapunov exponents (and thus D-bifurcation) in the Hopf Normal Form with Addative Noise. Our new approach is to study certain limits of the parameters to reduce the problem to previously studied toy models. The crucial ingredient for this kind of approach, is a result on the continuity of Lyapunov exponents. This is joint work with Maximilian Engel.