THE ISOPERIMETRIC INEQUALITY OUTSIDE CONVEX SETS

The starting point of this course is a classical isoperimetric inequality concerning the perimeter $P(E; H)$ of a set $E$ in a half space $H$. Namely, given $m > 0$, it is well known that among all sets $E \subset H$ of volume $m$ the minimizers of the perimeter of $E$ in $H$ are precisely the half balls of mass $m$ sitting on the hyperplane $\partial H$. Few years ago, this isoperimetric inequality was extended by Choe, Ghomi and Ritoré in [3] to the case of the relative perimeter of a set $E$ contained in the exterior of a convex set. The aim of this course is to present this general isoperimetric inequality together with a characterization of the equality cases recently obtained in [4].

The course is essentially self contained since most of the preliminary material will be presented in the lectures. However, a basic knowledge of Hausdorff measures, a good background in measure theory and some familiarity with the regularity theory of PDEs is required.

Lecture 1: Basic definitions and properties of sets of finite perimeter, coarea formula, De Giorgi’s structure theorem, the euclidean isoperimetric inequality.

Lecture 2: The relative isoperimetric inequality in a half space. First variation of volume and perimeter. The capillarity functional. Young’s law.


Lecture 4: The total curvature. An estimate of the total curvature of a set. A Willmore type inequality.


Lecture 6: Proof of the relative isoperimetric inequality outside a convex set.

Lecture 7: Characterization of the equality cases in the relative inequality outside a convex set.

REFERENCES


